Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0123456789

FURTHER MATHEMATICS **9231/01**

Paper 1 Further Pure Mathematics 1 **For examination from 2020**

SPECIMEN PAPER **2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 **(a)** Given that $f(r) = \frac{1}{(r+1)(r+2)}$, show that

$$f(r-1) - f(r) = \frac{2}{r(r+1)(r+2)}.$$ [2]

(b) Hence find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$. [3]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. [1]

2 It is given that $\phi(n) = 5^n(4n + 1) - 1$, for $n = 1, 2, 3, \ldots$.

Prove, by mathematical induction, that $\phi(n)$ is divisible by 8 for every positive integer n. [7]

3 The curve C has polar equation $r = 2 + 2\cos\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C. [3]

(b) Find the area of the region enclosed by C and the initial line. [4]

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(c) Show that the Cartesian equation of C can be expressed as $4(x^2 + y^2) = (x^2 + y^2 - 2x)^2$. [3]

4 The cubic equation

$$z^3 - z^2 - z - 5 = 0$$

has roots α, β and γ.

(a) Show that the value of $\alpha^3 + \beta^3 + \gamma^3$ is 19. [4]

(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4$. [2]

(c) Find a cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$, giving your answer in the form

$$px^3 + qx^2 + rx + s = 0,$$

where p, q, r and s are constants to be determined. [3]

[Turn over

5 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 5 & k \\ -3 & -4 \end{pmatrix}.$$

(a) Find the value of k for which **A** is singular. [2]

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It is now given that $k = 6$ so that $\mathbf{A} = \begin{pmatrix} 5 & 6 \\ -3 & -4 \end{pmatrix}$.

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **A**. [6]

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(c) The triangle *DEF* in the x-y plane is transformed by **A** onto triangle *PQR*.

(i) Given that the area of triangle *DEF* is $10\,\text{cm}^2$, find the area of triangle *PQR*. [2]

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(ii) Find the matrix which transforms triangle *PQR* onto triangle *DEF*. [2]

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6 The position vectors of the points A, B, C, D are

$$2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}, \quad -2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}, \quad \mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \mathbf{i} + 5\mathbf{j} + m\mathbf{k},$$

respectively, where m is an integer. It is given that the shortest distance between the line through A and B and the line through C and D is 3.

(a) Show that the only possible value of m is 2. [7]

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(b) Find the shortest distance of D from the line through A and C. [3]

(c) Show that the acute angle between the planes ACD and BCD is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$. [4]

7 The curve C has equation $y = \dfrac{2x^2 - 3x - 2}{x^2 - 2x + 1}$.

(a) State the equations of the asymptotes of C. [2]

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(b) Show that $y \leqslant \frac{25}{12}$ at all points on C. [4]

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(c) Find the coordinates of any stationary points of C. [3]

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(d) Sketch C, stating the coordinates of any intersections of C with the coordinate axes and the asymptotes. [4]

(e) Sketch the curve with equation $y = \left|\frac{2x^2 - 3x - 2}{x^2 - 2x + 1}\right|$ and find the set of values of x for which $\left|\frac{2x^2 - 3x - 2}{x^2 - 2x + 1}\right| < 2$. [4]

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Additional page

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